WARBSTOW CP SCHOOL

Warbstow C. P. School

Calculation Policy

Warbstow CP School has recently bought into Collins Busy Ant Maths programme. This is a mathematics course that ensures complete coverage of the 2014 Primary National Curriculum for Mathematics.

The course has at its core the following seven key principles:

1 To inspire **enjoyment of maths**
2 To assist in developing children's **conceptual understanding of maths**
3 To help **raise levels** of attainment for every child
4 To provide a **rigorous and cohesive scope and sequence** of the primary maths curriculum, while at the same time allowing for schools' own curriculum design
5 To promote the most **effective pedagogical methods** in the teaching of mathematics
6 To offer manageable strategies for **effective diagnostic, formative and summative assessment,** to inform planning and teaching
7 To strengthen the **home/school link**.

In addition to these seven key principles, Busy Ant Maths offers:

- a straightforward yet flexible approach to the teaching of mathematics
- lesson plans following a highly effective and proven lesson structure
- a weekly bank of practical hands-on Learning activities
- a detailed and systematic approach to the development of mental and written calculations
- extensive teacher support through materials which:
  - are sufficiently detailed to aid confidence
  - are rich enough to be varied and developed
  - take into account issues of pace and classroom management
  - give careful consideration to the key skill of appropriate and effective questioning
  - provide a careful balance of teacher intervention and pupil participation
  - encourage communication of methods and foster mathematical rigor
- controlled, manageable differentiation with activities and suggestions for at least three different ability groups in every lesson
- a stand-alone resource aimed at developing children's fluency in number facts, containing hundreds of whole-class, group, paired and individual games and activities
- pupil materials which are enjoyable and purposeful.

The Calculations Policy

This calculations policy has been written to provide schools with an understanding of when and how the four operations –addition, subtraction, multiplication and division, are taught in the Busy Ant Maths course*.

The aim of this policy is to offer schools an insight into coverage of the National curriculum 'Addition and subtraction' and 'Multiplication and division' domains in Busy Ant Maths. It is also designed to ensure consistency throughout the school and to make teachers aware of the continuity and progression in skill development across the year groups. It aims to enable staff, and parents, to see how the concepts, facts and calculation strategies and methods used in any particular year are taught, and how these build on previous learning and contribute to future learning.

The Busy Ant Maths course has been written in the belief that being able to calculate successfully is being able to:

- have a confident and competent understanding of numbers and the number system
- have instant recall of a set of basic number facts
- use a range of mental calculation strategies effectively, efficiently and flexibly
- use a range of written calculation methods accurately and appropriately
- use and apply all of the above in order to solve problems and reason mathematically.

Structure of the document

For each year group, years 1 to 6, the Busy Ant Maths Calculations Policy begins with an outline of the key knowledge and understanding of number and the number system, including place value that pupils are taught in Busy Ant Maths in order to calculate successfully.

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers up to 1000 in numerals and in words
- count from 0 in multiples of 1, 2, 3, 4, 5, 8, 10, 50 and 100, forwards and backwards

Taken from Year 3 Calculations Policy

Then, for each year group, a detailed summary is provided of how each of the four operations: addition, subtraction, multiplication and division – is taught in Busy Ant Maths. This summary includes information on the following:

Conceptual understanding and procedural fluency

The key concepts pupils need to know and understand in order to calculate successfully.

The number facts that pupils need to recall with fluency.

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- continue to recall and use addition facts to 20 fluently, and derive and use related facts up to 100, e.g. 130 + 50 = 180
- continue to add numbers mentally, including:
  - two two-digit numbers

Taken from Year 3 Calculations Policy

* This policy also reflects the pupil consolidation activities contained in Fluency in Number Facts, the Collins Maths series that accompanies the Busy Ant Maths course.

Reason mathematically and solve problems

The problem solving and reasoning skills pupils need to develop in order to use and apply their conceptual understanding and procedural fluency.

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve problems, including missing number problems, using number facts, place value, and more complex addition

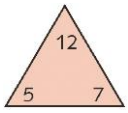
Taken from Year 3 Calculations Policy

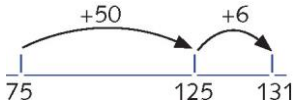
Mental strategies

The mental calculation strategies taught, including representations, and models and images used.

Mental strategies

- Use of models and images:
  - trios
    7 + 5 = 12
    5 + 7 = 12
    12 – 5 = 7
    12 – 7 = 5
  - the empty number line





Taken from Year 3 Calculations Policy

Written methods

The pencil and paper procedures taught – both informal and formal.

Written methods

- Add numbers with up to three digits (HTO + HTO)
- Estimate and check the answer to a calculation

Expanded written method

548 + 387

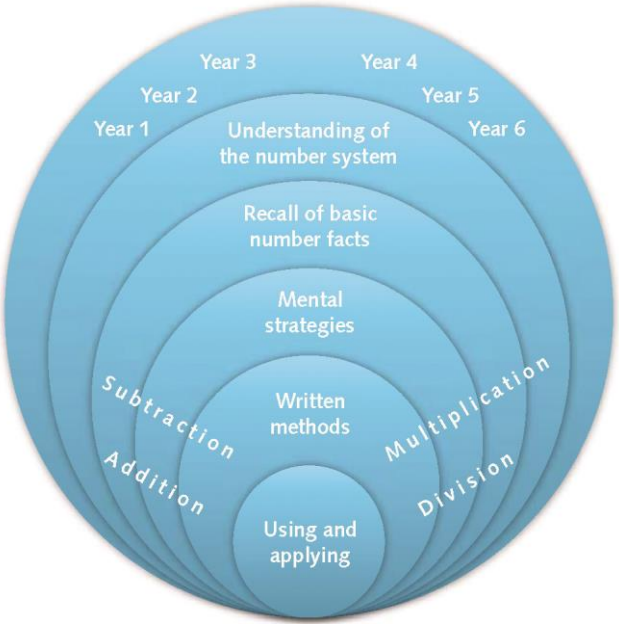
```
  548
+ 387
  ---
   15
  120
  800
  ---
  935
```

The first stage in the written method shows separately the addition of the ones to the ones, the tens to the tens and the hundreds to the hundreds. To find the partial sums either the ones or the hundreds can be added first, and the total of the

Taken from Year 3 Calculations Policy

The importance of making connections

Although the Busy Ant Maths Calculations Policy is organised by year group, then by operation, and finally by the areas outlined above, it is also vital to stress the importance of the interconnectivity between year groups, the four operations, and pupils' conceptual understanding, fluency in number facts, mental strategies and written methods, and their ability to reason mathematically and solve problems.



This calculations policy not only aims to show the continuity and progression of calculations within the Busy Ant Maths course, across the primary phase for each of the four operations, it also aims to emphasise the important link between the development of children's mental calculation strategies and the teaching of written calculation methods. Mental recall and strategies, and formal written methods must be seen as complementary to each other. Every written method has a component of mental processing so the two must constantly be developed in conjunction with each other. Pupils' mental facilities with number should be refined as they move through Key stage 2 and not focus exclusively on the written methods of calculation.

The ultimate goal of Busy Ant Maths, as for all primary schools, is to ensure that children are confident and competent in their calculation skills, and are able to use and apply these skills in the real world as autonomous problem solvers.

Year 1

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- count, read and write numbers from 1 to 20 in numerals and words
- count, read and write numbers to 100 in numerals
- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count in multiples of twos, fives and tens
- given a number, identify one more and one less
- compare and order numbers to at least 20
- identify and represent numbers using objects and pictorial representations, including the number line, and use the language of: equal to, more than, less than (fewer), most, least

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- understand addition as combining two or more groups of objects
- understand addition as counting on
- represent and use number bonds within 20
- add one-digit and two-digit numbers to 20, including zero
- realise the effect of adding zero
- recall doubles of all numbers to 10
- understand that addition can be done in any order
- read, write and interpret mathematical statements involving addition (+) and equals (=) signs

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

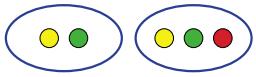
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 16 = ■ + 7 □
- solve one-step problems that involve addition in familiar contexts, e.g. money

Year 1

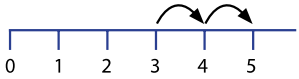
ADDITION Continued

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations



  - number tracks and number lines

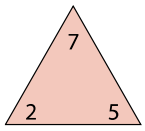


  - 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

  - trios

5 + 2 = 7
2 + 5 = 7
7 – 2 = 5
7 – 5 = 2



  - addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Identify near doubles, using doubles already known (e.g. 6 + 5)
- Recognise and use patterns of similar calculations (e.g. 10 + 0 = 10, 9 + 1 = 10, 8 + 2 = 10 ...)
- Understand and use the inverse relationship between addition and subtraction

Year 1

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- understand subtraction as 'taking away' (counting back)
- understand subtraction as 'finding the difference' (counting up)
- represent and use subtraction facts within 20
- subtract one-digit and two-digit numbers to 20, including zero
- realise the effect of subtracting zero
- understand that subtraction cannot be done in any order
- read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

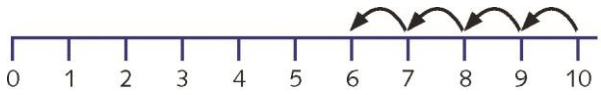
- solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = – 9 □
- solve one-step problems that involve subtraction in familiar contexts, e.g. money

Mental strategies

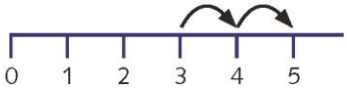
- Use of models and images:
  - concrete objects/pictorial representations



  - - number tracks and number lines: 'take away' (counting back)



'finding the difference' (counting up)

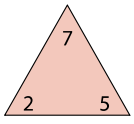


  - 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

  - trios

5 + 2 = 7
2 + 5 = 7
7 – 2 = 5
7 – 5 = 2



Year 1

SUBTRACTION Continued

Mental strategies continued

- addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Recognise and use patterns of similar calculations (e.g. 10 – 0 = 10, 10 – 1 = 9, 10 – 2 = 8)
- Understand and use the inverse relationship between addition and subtraction

Year 1

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- understand multiplication through grouping small quantities
- understand the link between multiplication and doubling

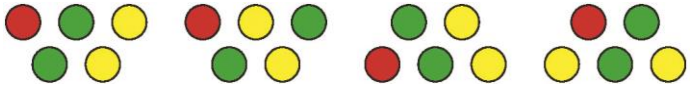
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- solve one-step problems that involve multiplication in familiar contexts

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations



  - arrays



4 lots of 5 is 20

5 lots of 4 is 20

- Make connections between arrays, number patterns and counting in steps of a constant size

Year 1

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- understand division through sharing small quantities
- understand the link between division and halving

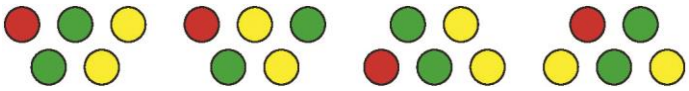
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- solve one-step problems that involve division in familiar contexts

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations



  - arrays



4 lots of 5 is 20

5 lots of 4 is 20

- Make connections between arrays, number patterns and counting in steps of a constant size

Year 2

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers to at least 100 in numerals and in words
- count in steps of 2, 3, and 5 from 0, and in tens from any number, forwards and backwards
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use <, > and = signs

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- recall and use addition facts to 20 fluently, and derive and use related facts up to 100, including adding two multiples of 10, e.g. 30 + 50
- add numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
- record addition in columns to support place value and prepare for the formal written method with larger numbers

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

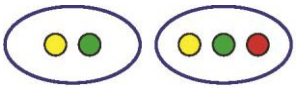
- solve problems with addition:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods

Year 2

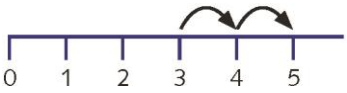
ADDITION Continued

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations



  - number tracks and number lines

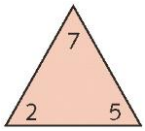


  - 1–100 number square

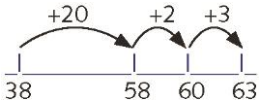
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

  - trios

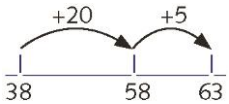
5 + 2 = 7
2 + 5 = 7
7 – 2 = 5
7 – 5 = 2



  - empty number line



leading to:



  - addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Use knowledge that addition can be done in any order (commutative), e.g.
  - put the larger number first and count on in tens or ones
  - add three small numbers by putting the largest number first and/or find a pair totalling 10

Year 2

ADDITION Continued

Mental strategies continued

- Partition additions into tens and ones, then recombine, e.g.

38 + 25 = 30 + 20 + 8 + 5
= 50 + 13
= 63

38 + 25 = 38 + 20 + 5
= 58 + 5
= 63

38 + 25

30 + 8
\+ 20 + 5
50 + 13 = 63

- Identify near doubles, using doubles already known (e.g. 7 + 8, 30 + 31)
- Add a 'near multiple of 10' to a two-digit number by adding 10, 20, 30 and adjusting
- Recognise and use patterns of similar calculations (e.g. 10 + 0 = 10, 9 + 1 = 10, 8 + 2 = 10 ...)
- Understand and use the inverse relationship between addition and subtraction

Written methods

- Add two two-digit numbers: TO + TO (where answers do not exceed 100)

Expanded written method

38 + 25

38
\+ 25
13
50
63

Record addition calculations in columns to support place value and prepare for the formal written method of columnar addition with larger numbers.

The first stage in the written method shows separately the addition of the ones to the ones and the tens to the tens. To find the partial sums either the ones or the tens can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method.

The addition of the tens in the calculation 38 + 25 is described in the words 'thirty add twenty equals fifty', stressing the link to the related fact 'three add two equals five'.

Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

Year 2

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100, including subtracting two multiples of 10, e.g. 80 – 30
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
- record subtraction in columns to support place value and prepare for the formal written method with larger numbers

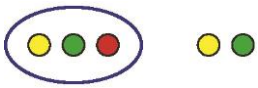
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

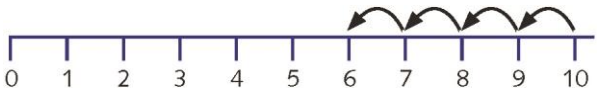
- solve problems with subtraction:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods

Mental strategies

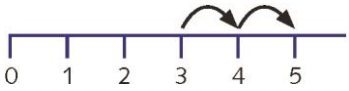
- Use of models and images:
  - concrete objects/pictorial representations



  - number tracks and number lines: 'take away' (counting back)



'finding the difference' (counting up)



  - 1–100 number square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

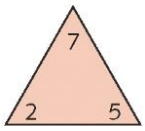
Year 2

SUBTRACTION Continued

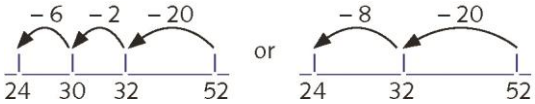
Mental strategies continued

- trios

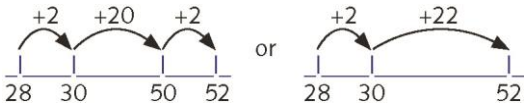
5 + 2 = 7
2 + 5 = 7
7 – 2 = 5
7 – 5 = 2



- empty number line: 'take away' (counting back)



'finding the difference' (counting up)



- addition and subtraction tables

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

+	11	12	13	14	15	16	17	18	19	20
0	11	12	13	14	15	16	17	18	19	20
1	12	13	14	15	16	17	18	19	20	
2	13	14	15	16	17	18	19	20		
3	14	15	16	17	18	19	20			
4	15	16	17	18	19	20				
5	16	17	18	19	20					
6	17	18	19	20						
7	18	19	20							
8	19	20								
9	20									

- Find a small difference by counting up from the smaller to the larger number, e.g. 51 – 4
- Subtract a 'near multiple of 10' from a two-digit number by subtracting 10, 20, 30 and adjusting
- Recognise and use patterns of similar calculations (e.g. 10 – 0 = 10, 10 – 1 = 9, 10 – 2 = 8 ...)
- Understand and use the inverse relationship between addition and subtraction
- Use partitioning, e.g.

52 – 28 = 52 – 20 – 8
= 32 – 8
= 24

Written methods

- Subtract two two-digit numbers: TO – TO (that do not require decomposition)

87 – 32

$$\begin{array}{r} 87 \\ -\ 32 \\ \hline 55 \end{array}$$

Record subtraction calculations that do not require decomposition in columns to support place value and prepare for formal written methods of columnar subtraction with larger numbers.

Where appropriate, place value columns are labelled, e.g. TO, to remind children of the value of each of the digits.

Year 2

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- recognise multiplication as repeated addition
- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs

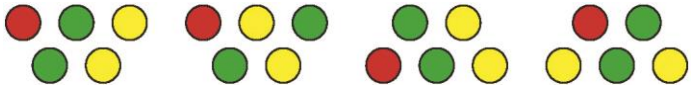
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations



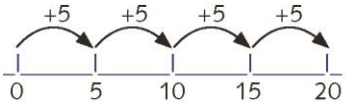
  - arrays

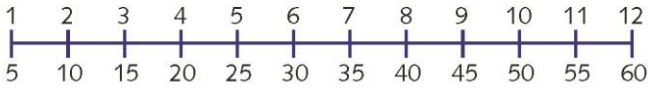


4 × 5 = 20

5 × 4 = 20

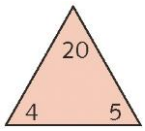
  - number lines





  - trios

4 × 5 = 20
5 × 4 = 20
20 ÷ 5 = 4
20 ÷ 4 = 5



  - multiplication and division table

×	2	5	10
1	2	5	10
2	4	10	20
3	6	15	30
4	8	20	40
5	10	25	50
6	12	30	60
7	14	35	70
8	16	40	80
9	18	45	90
10	20	50	100
11	22	55	110
12	24	60	120

Year 2

MULTIPLICATION Continued

Mental strategies continued

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division, including doubling and halving

Year 2

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- recognise division as grouping or sharing
- recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- understand the link between division and fractions, and find fractions of a length, shape, set of objects or quantity
- calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs

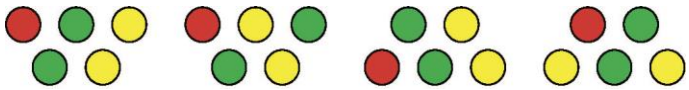
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems involving division, using materials, arrays, repeated addition and subtraction, mental methods, and division facts, including problems in contexts

Mental strategies

- Use of models and images:
  - concrete objects/pictorial representations

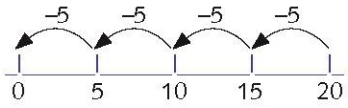


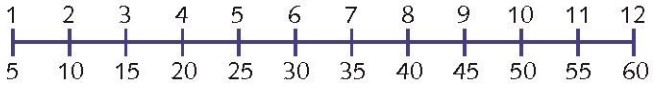
  - arrays



20 ÷ 5 = 4
20 ÷ 4 = 5

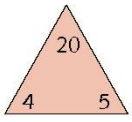
  - number lines





  - trios

4 × 5 = 20
5 × 4 = 20
20 ÷ 5 = 4
20 ÷ 4 = 5



Year 2

DIVISION Continued

Mental strategies continued

- multiplication and division table

×	2	5	10
1	2	5	10
2	4	10	20
3	6	15	30
4	8	20	40
5	10	25	50
6	12	30	60
7	14	35	70
8	16	40	80
9	18	45	90
10	20	50	100
11	22	55	110
12	24	60	120

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division, including doubling and halving

Year 3

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers up to 1000 in numerals and in words
- count from 0 in multiples of 1, 2, 3, 4, 5, 8, 10, 50 and 100, forwards and backwards
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- identify, represent and estimate numbers using different representations
- find 10 or 100 more or less than a given number
- compare and order numbers up to 1000

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- continue to recall and use addition facts to 20 fluently, and derive and use related facts up to 100, e.g. 130 + 50 = 180
- continue to add numbers mentally, including:
  - two two-digit numbers
  - three or more one-digit numbers
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add numbers with up to three digits, using the formal written method of columnar addition
- estimate and check the answer to a calculation, including using the inverse operation

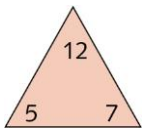
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

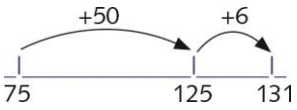
- solve problems, including missing number problems, using number facts, place value, and more complex addition

Mental strategies

- Use of models and images:
  - Trios 7 + 5 = 12
5 + 7 = 12
12 – 5 = 7
12 – 7 = 5



  - the empty number line



Year 3

ADDITION Continued

Mental strategies continued

- To multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Recognise and use the inverse relationship between addition and subtraction
- Use knowledge that addition can be done in any order (commutative), e.g.
  - put the larger number first and count on in steps of 1, 10 or 100
  - partition additions into hundreds, tens and ones, then recombine, e.g. 75 + 56 = 75 + 50 + 6
    = 125 + 6
    = 131
- Identify near doubles, using doubles already known, e.g. 70 + 71
- Add the nearest multiple of 10 or 100, and adjust
- Use patterns of similar calculations, e.g. 13 + 5 = 18 and 130 + 50 = 180
- Use knowledge of the associative law when adding more than two numbers, e.g. 4 + 7 + 6 = (4 + 6) + 7
  = 10 + 7
  = 17

Year 3

ADDITION Continued

Written methods

- Add numbers with up to three digits (HTO + HTO)
- Estimate and check the answer to a calculation

Expanded written method

548+387

```
  548
+ 387
  15
 120
 800
 935
```

The first stage in the written method shows separately the addition of the ones to the ones, the tens to the tens and the hundreds to the hundreds. To find the partial sums either the ones or the hundreds can be added first, and the total of the partial sums can be found by adding them in any order. Children should be encouraged to start by adding the ones digits first (the least significant digits), as this echoes the formal written method of columnar addition.

The addition of the tens in the calculation 548 + 387 is described in the words 'forty add eighty equals one hundred and twenty', stressing the link to the related fact 'four add eight equals twelve'. The addition of the hundreds is described as 'five hundred add three hundred equals eight hundred', stressing the link to the related fact 'five add three equals eight'.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of columnar addition

548+387

```
  548
+ 387
  935
   11
```

The expanded written method leads to the formal written method of columnar addition so that children fully understand the procedure, and the effectiveness and efficiency of the method.

Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 3

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- continue to recall and use subtraction facts to 20 fluently, and derive and use related facts, e.g. 120 – 70 = 50
- continue to subtract numbers mentally, including:
  - two two-digit numbers
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- subtract numbers with up to three digits, using the formal written method of columnar subtraction
- estimate and check the answer to a calculation, including using the inverse operation

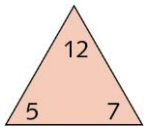
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

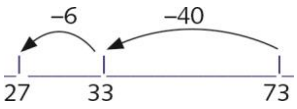
- solve problems, including missing number problems, using number facts, place value, and more complex subtraction

Mental strategies

- Use of models and images:
  - trios
    7 + 5 = 12
    5 + 7 = 12
    12 – 5 = 7
    12 – 7 = 5



  - the empty number line



  - multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Recognise and use the inverse relationship between addition and subtraction
- Find a difference by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10 or 100, and adjust
- Use patterns of similar calculations, e.g. 18 – 5 = 13 and 180 – 50 = 130
- Use partitioning, e.g. 73 – 46 = 73 – 40 – 6
  = 33 – 6
  = 27

Year 3

SUBTRACTION Continued

Written methods

- Subtract numbers with up to three digits (HTO – HTO)
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

582 – 237

```
   ~~5~~ ~~8~~ 12
     5  8  2
–    2  3  7
   3  4  5
```

Start by subtracting the least significant digits first, i.e. the ones, then the tens, and finally the hundreds. Refer to subtracting the tens, for example, by saying 'seventy subtract thirty', not 'seven subtract three'.

In this example the ones to be subtracted are larger than the ones you are subtracting from.

You exchange one of the 8 tens for 10 ones, crossing out 8 and writing a superscript 7, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 subtract 7, 70 subtract 30 and finally 500 subtract 200.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 3

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of multiplication facts for the 2, 5 and 10 multiplication tables
- recall and use multiplication facts for the 3, 4 and 8 multiplication tables
- use known multiplication facts to derive related facts involving multiples of 10, e.g. 2 × 30 = 60
- write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods and progressing to formal written methods
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

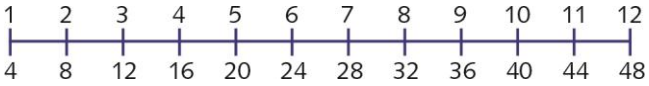
Mental strategies

- Use of models and images:
  - arrays



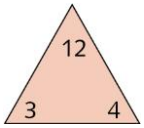
3 × 4 = 12
4 × 3 = 12

  - number lines



  - trios

3 × 4 = 12
4 × 3 = 12
12 ÷ 4 = 3
12 ÷ 3 = 4



  - multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division
- Use doubling, e.g. connect the 2, 4 and 8 multiplication tables

Year 3

MULTIPLICATION Continued

Mental strategies continued

- Use the 'key multiplication facts' of x 1, x 2, x 5, and x 10 to work out the answers to unknown multiplication facts, e.g. 7 × 4 = (5 × 4) + (2 × 4)
  = 20 + 8
  = 28
- Use patterns of similar calculations, e.g. 8 × 6 = 48 and 8 × 60 = 480
- Show that multiplication of two numbers can be done in any order (commutative), e.g. 4 × 12 × 5 = 4 × 5 × 12
  = 20 × 12
  = 240
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit number by a one-digit number, 63 × 8 = (60 × 8) + (3 × 8)
  = 480 + 24
  = 504

Written methods

- Short multiplication:
  - Multiply a two-digit number by a one-digit number (TO x O)
- Estimate and check the answer to a calculation

Grid method

63 × 8

×	60	3
8	480	24

= 504

Expanded written method

63 × 8

6 3
× 8
2 4 (3 × 8)
4 8 0 (60 × 8)
5 0 4
1

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 63 × 8 it is 'sixty multiplied by eight', not 'six multiplied by eight', although the relationship 6 × 8 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

6 3
× 2 8
5 0 4

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 3

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of division facts for the 2, 5 and 10 multiplication tables
- recall and use division facts for the 3, 4 and 8 multiplication tables
- use known division facts to derive related facts involving multiples of 10, e.g. 60 ÷ 3 = 20
- write and calculate mathematical statements for division using the multiplication tables that they know
- develop reliable written methods for division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written method of short division (with a remainder)
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems, including missing number problems, involving division

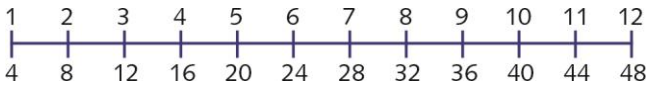
Mental strategies

- Use of models and images:
  - arrays



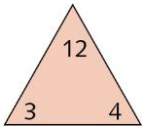
12 ÷ 4 = 3
12 ÷ 3 = 4

  - number lines



  - trios

3 × 4 = 12
4 × 3 = 12
12 ÷ 4 = 3
12 ÷ 3 = 4



  - multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

- Make connections between arrays, number patterns and counting in steps of a constant size
- Understand and use the inverse relationship between multiplication and division
- Use halving, e.g. find quarters by halving halves

Year 3

DIVISION Continued

Mental strategies continued

- Use patterns of similar calculations, e.g. 48 ÷ 8 = 6 and 480 ÷ 80 = 6
- Understand and use the distributive law, e.g. partitioning when dividing a two-digit number by a one-digit number,

92 ÷ 4 = (80 ÷ 4) + (12 ÷ 4)
= 20 + 3
= 23

Written methods

- Short division (without a remainder):
  - Divide a two-digit number by a one-digit number (TO ÷ O)
- Estimate and check the answer to a calculation

Expanded written method

92 ÷ 4

```
  2 3
4)9 2
  8 0   ( 20 × 4)
  1 2
  1 2   (  3 × 4)
    0
```

The first step is to show all of the calculations involved.

Children should describe what they are doing using phrases similar to the following: 'How many fours divide into 90 so that the answer is a multiple of 10? (20) There are 20 fours or 80, with 12 remaining. How many fours in 12? (3) So 92 divided by four is 23.'

Formal written method of short division

```
  2 3
4)9 12
```

The expanded written method leads children to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 1 represents the 1 ten that is remaining after 4 has been divided into 90. It is written in front of the 2 to show that 12 now has to be divided by 4.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Year 4

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read and write numbers up to 10 000 in numerals and in words
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- identify, represent and estimate numbers using different representations
- find 10, 100 or 1000 more or less than a given number
- compare and order numbers beyond 1000
- round any number to the nearest 10, 100 or 1000

DECIMALS

To add and subtract successfully, pupils need to:

- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise the place value of each digit in a decimal to two decimal places
- compare and order numbers with the same number of decimal places up to two decimal places
- round decimals with one decimal place to the nearest whole number
- understand the effect of multiplying and dividing a one-digit or two-digit number by 10 and 100

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- consolidate recall of addition facts to 20 and related facts involving multiples of 100 and 1000, e.g. 1300 + 500 = 1800 and 500 + 1300 = 1800
- continue to add numbers mentally, including:
  - two two-digit numbers
  - three or more one-digit numbers
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add numbers with up to four digits using the formal written method of columnar addition where appropriate, including calculations involving money, e.g. £13.56 + £38.54
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

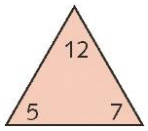
- solve addition two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving decimals to two decimal places

Year 4

ADDITION Continued

Mental strategies

- Continue to use models and images when necessary:
  - trios
    7 + 5 = 12
    5 + 7 = 12
    12 – 5 = 7
    12 – 7 = 5



  - multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Continue to use the relationship between addition and subtraction
- Use knowledge of the commutative law, e.g.
  - put the larger number first and count on in steps of 1, 10 or 100
  - partition additions into hundreds, tens and ones, then recombine, e.g. 356 + 57 = 356 + 50 + 7
    = 406 + 7
    = 413
- Identify near doubles, using doubles already known, e.g. 170 + 180
- Add the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 130 + 5 = 180 and 1300 + 500 = 1800
- Use knowledge of the associative law when adding more than two numbers, e.g. 24 + 27 + 16 = (24 + 16) + 27
  = 40 + 27
  = 67

Written methods

- Add numbers with up to four digits, including money and measures (ThHTO + ThHTO)
- Estimate and check the answer to a calculation

Formal written method of columnar addition

2456 + 5378

```
  2 4 5 6
+ 5 3 7 8
-------
  7 8 3 4
  1 1
```

Carry digits are recorded below the line, using the words 'carry ten', 'carry one hundred', or 'carry one thousand', not 'carry one'.
Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.
If necessary, remind children of the expanded written method so that they fully understand the procedure, and the effectiveness and efficiency of the formal written method of columnar addition.

Year 4

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- consolidate recall of subtraction facts to 20 and related facts involving multiples of 100 and 1000, e.g. 1800 – 500 = 1300 and 1800 – 1300 = 500
- continue to subtract numbers mentally, including:
  - two two-digit numbers
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- subtract numbers with up to four digits using the formal written method of columnar subtraction where appropriate, including calculations involving money, e.g. £24.26 – £17.58
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

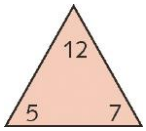
Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve subtraction two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving decimals to two decimal places

Mental strategies

- Continue to use models and images when necessary:
  - trios

    7 + 5 = 12
    5 + 7 = 12
    12 – 5 = 7
    12 – 7 = 5



  - multiples of 10 addition and subtraction tables

+	0	10	20	30	40	50	60	70	80	90	100
0	0	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100	110
20	20	30	40	50	60	70	80	90	100	110	120
30	30	40	50	60	70	80	90	100	110	120	130
40	40	50	60	70	80	90	100	110	120	130	140
50	50	60	70	80	90	100	110	120	130	140	150
60	60	70	80	90	100	110	120	130	140	150	160
70	70	80	90	100	110	120	130	140	150	160	170
80	80	90	100	110	120	130	140	150	160	170	180
90	90	100	110	120	130	140	150	160	170	180	190
100	100	110	120	130	140	150	160	170	180	190	200

+	110	120	130	140	150	160	170	180	190	200
0	110	120	130	140	150	160	170	180	190	200
10	120	130	140	150	160	170	180	190	200	210
20	130	140	150	160	170	180	190	200	210	220
30	140	150	160	170	180	190	200	210	220	230
40	150	160	170	180	190	200	210	220	230	240
50	160	170	180	190	200	210	220	230	240	250
60	170	180	190	200	210	220	230	240	250	260
70	180	190	200	210	220	230	240	250	260	270
80	190	200	210	220	230	240	250	260	270	280
90	200	210	220	230	240	250	260	270	280	290
100	210	220	230	240	250	260	270	280	290	300

- Continue to use the relationship between addition and subtraction
- Calculate mentally a difference such as 5005 – 2998 by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 18 – 5 = 13 and 1800 – 500 = 1300
- Use partitioning, e.g. 456 – 84 = 456 – 80 – 4
  = 376 – 4
  = 372

Year 4

SUBTRACTION Continued

Written methods

- Subtract numbers with up to four digits, including money and measures (ThHTO – ThHTO)
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

6418 – 2546

```
  5 13 11
  6 4 1 8
– 2 5 4 6
  3 8 7 2
```

Start by subtracting the least significant digits first, i.e. the ones, then the tens, then the hundreds and finally the thousands. Refer to subtracting the tens, for example, by saying '11 tens subtract four tens', not '11 subtract four'.

In this example the tens and the hundreds to be subtracted are larger than both the tens and hundreds you are subtracting from.

The calculation begins 8 subtract 6.

Then you exchange one of the 4 hundreds for 10 tens, crossing out 4 and writing a superscript 3, and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 tens subtract 4 tens.

You then exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5, and writing a superscript 1 in front of the 3 to make 13 hundreds. The calculation then becomes 13 hundreds subtract 5 hundreds.

Then finally 5000 subtract 2000.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Year 4

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables
- recall and use multiplication facts for the 6, 7, 9, 11 and 12 multiplication tables
- use known multiplication facts to derive related facts involving multiples of 10 and 100, e.g. 200 x 3 = 600
- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems involving multiplying and adding, including using the distributive law and multiply two-digit numbers by one digit, including scaling problems and harder correspondence problems such as n objects are connected to m objects

Mental strategies

- Continue to use models and images:
  - arrays 

    6 × 7 = 42

    7 × 6 = 42
  - number lines

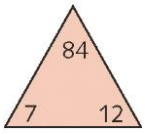
    1 2 3 4 5 6 7 8 9 10 11 12

    12 24 36 48 60 72 84 96 108 120 132 144
  - trios 7 × 12 = 84

    12 × 7 = 84

    84 ÷ 12 = 7

    84 ÷ 7 = 12

  - multiplication square to 12 × 12

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

- Make connections between arrays, number patterns and counting in steps of a constant size

Year 4

MULTIPLICATION Continued

Mental strategies continued

- Continue to use the inverse relationship between multiplication and division
- Continue to use doubling, e.g. connect the 3, 6 and 12 multiplication tables
- Use the 'key multiplication facts' of × 1, × 2, × 5, and × 10 to work out the answers to unknown multiplication facts,
  e.g. 7 × 9 = (5 × 9) + (2 × 9)
  = 45 + 18
  = 63
- Use closely related facts:
  - multiply by 9 or 11 by multiplying by 10 and adjusting
  - develop the × 12 table by adding facts from the × 10 and × 2 table
- Use factors, e.g. 8 x 14 = 8 × 2 × 7
- Use patterns of similar calculations, e.g. 8 × 6 = 48 and 8 × 60 = 480
- Understand and use the commutative law
- Understand and use the associative law, e.g. 6 × 15 = 6 × (5 × 3)
  = (6 × 5) × 3
  = 30 × 3
  = 90
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, e.g.
  356 × 7 = (300 × 7) + (50 × 7) + (6 × 7)
  = 2100 + 350 + 42
  = 2492

Written methods

- Short multiplication:
  - Multiply a two-digit or three-digit number by a one-digit number (TO × O/HTO × O)
- Estimate and check the answer to a calculation

Grid method

356 × 7

×	300	50	6	
7	2100	350	42	= 2492

Expanded written method

356 × 7

```
  3 5 6
×     7
-------
    4 2   (  6 × 7)
  3 5 0   ( 50 × 7)
2 1 0 0   (300 × 7)
-------
  2 4 9 2
```

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 356 × 7 it is 'fifty multiplied by seven', not 'five multiplied by seven', although the relationship 5 × 7 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

356 × 7

```
  3 5 6
×   3 4 7
---------
  2 4 9 2
```

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.
The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.
Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 4

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables
- recall and use division facts for the 6, 7, 9, 11 and 12 multiplication tables
- use known division facts to derive related facts involving multiples of 10 and 100, e.g. 600 ÷ 3 = 200
- use place value, known and derived facts to divide mentally, including dividing by 1
- recognise and use factor pairs in mental calculations
- divide two-digit and three-digit numbers by a one-digit number using formal written layout (without a remainder)
- estimate and check the answer to a calculation, including using the inverse operation

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems involving division in contexts, deciding which operations and methods to use and why

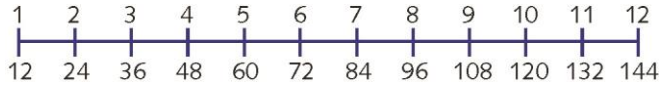
Mental strategies

- Continue to use models and images:
  - arrays



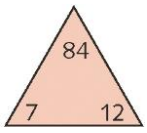
42 ÷ 7 = 6
42 ÷ 6 = 7

  - number lines



  - trios

7 × 12 = 84
12 × 7 = 84
84 ÷ 12 = 7
84 ÷ 7 = 12



  - multiplication square to 12 × 12

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

- Make connections between arrays, number patterns and counting in steps of a constant size
- Continue to use the inverse relationship between multiplication and division
- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables

Year 4

DIVISION Continued

Mental strategies continued

- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number,
  486 ÷ 9 = (450 ÷ 9) + (36 ÷ 9)
  = 50 + 4
  = 54

Written methods

- Short division (without a remainder):
  - Divide a two-digit or three-digit number by a one-digit number (TO ÷ O/HTO ÷ O)
- Estimate and check the answer to a calculation

Expanded written method

486 ÷ 9

```
      5 4
   ┌──────
9) 4 8 6
   4 5 0   (50 × 9)
   ─────
     3 6
     3 6   (4 × 9)
   ─────
       0
```

The first step is to show all of the calculations involved.

Children should describe what they are doing using phrases similar to the following: 'How many nines divide into 480 so that the answer is a multiple of 10? (50) There are 50 nines or 450, with 36 remaining. How many nines in 36? (4) So 486 divided by nine is 54.'

Formal written method of short division

```
    5 4
   ──────
9) 4 8³6
```

The expanded written method leads to the formal written method of short division so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The superscript 3 represents the 3 tens that are remaining after 9 has been divided into 480. It is written in front of the 6 to show that 36 now has to be divided by 9.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Year 5

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- find 10, 100, 1000, 10 000 or 100 000 more or less than a given number
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000

DECIMALS

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers with up to three decimal places
- identify the value of each digit in numbers given to three decimal places
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- add numbers mentally with increasingly large numbers
- add decimals, including a mix of one-digit whole numbers and tenths, e.g. 1·3 + 0·5; and complements of 1, e.g. 0·83 + 0·17
- add whole numbers with more than four digits, including using the formal written method (columnar addition)
- add decimals to two places, including using the formal written method (columnar addition)
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving number up to three decimal places

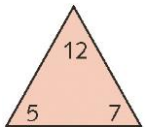
Year 5

ADDITION Continued

Mental strategies

- Continue to use models and images when necessary:
  - trios

7 + 5 = 12
5 + 7 = 12
12 – 5 = 7
12 – 7 = 5



  - decimals addition and subtraction tables

+	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0·1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1
0·2	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
0·3	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3
0·4	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4
0·5	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5
0·6	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6
0·7	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7
0·8	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8
0·9	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9
1	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2

+	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0·01	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11
0·02	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12
0·03	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13
0·04	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14
0·05	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15
0·06	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16
0·07	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17
0·08	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18
0·09	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19
0·1	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19	0·2

- Develop further the relationship between addition and subtraction
- Continue to apply knowledge of the commutative law, e.g.
  - put the larger number first and count on in steps of 1, 10, 100 or 1000;
  - partition additions into hundreds, tens and ones, then recombine, e.g. 356 + 57 = 356 + 50 + 7
    = 406 + 7
    = 413
- Identify near doubles, using doubles already known, e.g. 1·7 + 1·8
- Add the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 9 + 7 = 16 and 0·9 + 0·7 = 1·6
- Use knowledge of the associative law when adding more than two numbers, e.g. 24 + 27 + 16 = (24 + 16) + 27
  = 40 + 27
  = 67

Written methods

- Add whole numbers with more than four digits
- Add decimals with up to two decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar addition

12 957 + 14 635

```
  1 2 9 5 7
+ 1 4 6 3 5
-----------
  2 7 5 9 2
    1   1
```

56·47 + 84·84

```
    5 6 · 4 7
+   8 4 · 7 6
-------------
  1 4 1 · 3 3
    1 1 1
```

Carry digits are recorded below the line, using the words 'carry ten', 'carry one hundred', 'carry one thousand' or 'carry ten thousand', not 'carry one'.

Where appropriate, place value columns are labelled, e.g. TO·th, to remind children of the value of each of the digits.

Year 5

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- subtract numbers mentally with increasingly large numbers
- subtract decimals, including a mix of one-digit whole numbers and tenths, e.g. 1·8 – 0·7
- subtract whole numbers with more than four digits, including using the formal written method (columnar subtraction)
- subtract decimals to two places, including using the formal written method (columnar subtraction)
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

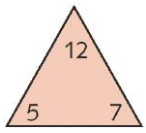
Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving numbers up to three decimal places

Mental strategies

- Continue to use models and images when necessary:
  - trios

7 + 5 = 12
5 + 7 = 12
12 – 5 = 7
12 – 7 = 5



  - decimals addition and subtraction tables

+	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0·1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1
0·2	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
0·3	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3
0·4	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4
0·5	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5
0·6	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6
0·7	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7
0·8	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8
0·9	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9
1	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2

+	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0·01	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11
0·02	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12
0·03	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13
0·04	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14
0·05	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15
0·06	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16
0·07	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17
0·08	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18
0·09	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19
0·1	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19	0·2

- Develop further the relationship between addition and subtraction
- Calculate mentally a difference such as 12 462 – 2300 by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 16 – 9 = 7 and 1·6 – 0·9 = 0·7
- Use partitioning, e.g. 456 – 84 = 456 – 80 – 4
  = 376 – 4
  = 372

Year 5

SUBTRACTION Continued

Written methods

- Subtract whole numbers with more than four digits
- Subtract decimals with up to two decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

45 257 – 17 488

```
  3 14 11 14 17
  4 5 2 5 7
– 1 7 4 8 8
  2 7 7 6 9
```

83·72 – 36·49

```
  7 13   6 12
  8 3 · 7 2
– 3 6 · 4 9
  4 7 · 2 3
```

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens, then the hundreds, then the thousands and finally the tens of thousands. Refer to subtracting the tens, for example, by saying '14 tens subtract 8 tens', not '14 subtract 8'.

In the first example the ones, tens, hundreds and thousands to be subtracted are all larger than all of the ones, tens, hundreds and thousands you are subtracting from.

The calculation begins by exchanging one of the 5 tens for 10 ones, crossing out the 5 and writing a superscript 4, and crossing out the 7 and writing a superscript 17. The calculation then becomes 17 subtract 8.

You then exchange one of the 2 hundreds for 10 tens, crossing out the 2 and writing a superscript 1, and writing a superscript 1 in front of the 4 to make 14 tens. The calculation then becomes 14 tens subtract 8 tens.

Next, you exchange one of the 5 thousands for 10 hundreds, crossing out the 5 and writing a superscript 4, and writing a superscript 1 in front of the 1 to make 11 hundreds. The calculation then becomes 11 hundreds subtract 4 hundreds.

Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 4 to make 14 thousands. The calculation then becomes 14 thousands subtract 7 thousands.

Then finally 30 000 subtract 10 000.

Where appropriate, place value columns are labelled, e.g. TO·th, to remind children of the value of each of the digits.

Year 5

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of the multiplication facts for multiplication tables up to 12 × 12
- use known multiplication facts to derive related facts involving multiples of 10, 100 and 1000, e.g. 70 × 80 = 5600
- continue to use place value, known and derived facts to multiply mentally
- multiply whole numbers and those involving decimals by 10, 100 and 1000
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- continue to recognise commutativity in mental calculations
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems, involving multiplication including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates

Year 5

MULTIPLICATION Continued

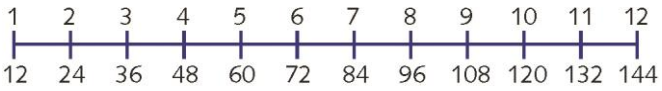
Mental strategies

- Continue to use models and images when necessary:
  - arrays



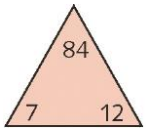
6 × 7 = 42
7 × 6 = 42

  - number lines



  - trios

7 × 12 = 84
12 × 7 = 84
84 ÷ 12 = 7
84 ÷ 7 = 12



  - multiplication square to 12 × 12/multiples of 10 multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

×	20	30	40	50	60	70	80	90	100	110	120
1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	240
3	60	90	120	150	180	210	240	270	300	330	360
4	80	120	160	200	240	280	320	360	400	440	480
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

- Continue to use the inverse relationship between multiplication and division
- Use related facts and doubling and halving:
  - double or halve the most significant digit first
  - to multiply by 25, multiply by 100 then divide by 4
  - double one number and halve the other
  - find the multiplication facts for the × 16 multiplication table by doubling the × 8 multiplication facts
- Use closely related facts:
  - multiply by 19 or 21 by multiplying by 20 and adjusting
  - develop the × 14 multiplication table by adding facts from the × 12 and × 2 multiplication tables
- Use factors, e.g. 9 x 18 = 9 × 6 × 3
- Use patterns of similar calculations, e.g. 8 × 6 = 48 and 8 × 600 = 4800
- Understand and use the commutative law, e.g. 14 × 12 = (2 × 7) × 12
  = 2 × (7 × 12)
  = 2 × 84
  = 168
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one digit number, or two two-digit numbers, e.g.

378 × 4 = (300 × 4) + (70 × 4) + (8 × 4)
= 1200 + 280 + 32
= 1512

78 × 34 = (78 × 30) + (78 × 4)
= 2340 + 312
= 2652

Year 5

MULTIPLICATION Continued

Written methods

- Short multiplication:
  - Multiply numbers up to four digits by a one-digit number (HTO × O/ThHTO × O)
- Estimate and check the answer to a calculation

Grid method

378 × 4

×	300	70	8	
4	1200	280	32	= 1512

Expanded written method

378 × 4

$$\begin{array}{rl} 378 & \\ \times\ 4 & \\ \hline 32 & (8 \times 4) \\ 280 & (70 \times 4) \\ 1200 & (300 \times 4) \\ \hline 1512 & \\ 1 & \end{array}$$

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns (e.g. when multiplying the tens in 378 × 4 it is 'seventy multiplied by four', not 'seven multiplied by four', although the relationship 7 × 4 should be stressed).

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

378 × 4

$$\begin{array}{r} 378 \\ \times\ 34 \\ \hline 1512 \end{array}$$

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

- Long multiplication:
  - Multiply numbers up to three digits by a two-digit number (TO × TO/HTO × TO)
- Estimate and check the answer to a calculation

Grid method

78 × 44

×	70	8	
30	2100	240	2340 →
4	280	32	+ 312 →
			2652 →

$$\begin{array}{rl} 78 & \\ \times\ 34 & \\ \hline 2340 & (78 \times 30) \\ 312 & (78 \times 4) \\ \hline 2652 & \end{array}$$

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 78 is multiplied by 30 (using knowledge of 78 x 3), then 78 is multiplied by 4, and finally the two products are added together.

Year 5

MULTIPLICATION Continued

Written methods continued

Expanded written method

78 × 34

Multiplying the most significant digit first

$$\begin{array}{r} 78 \\ \times\ 34 \\ \hline 2\,3^{2}4\,0 \quad (78 \times 30) \\ 3\,1^{3}\,2 \quad (78 \times 4) \\ \hline 2652 \end{array}$$

Multiplying the least significant digit first

$$\begin{array}{r} 78 \\ \times\ 34 \\ \hline 312 \quad (78 \times 4) \\ 2340 \quad (78 \times 30) \\ \hline 2652 \end{array}$$

The grid method leads to the expanded written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

Year 5

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of the division facts for multiplication tables up to 12 × 12
- use known division facts to derive related facts involving multiples of 10, 100 and 1000, e.g. 6300 ÷ 90 = 70
- continue to use place value, known and derived facts to divide mentally
- divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- use rounding to estimate and check answers to calculations and determine, in the context of a problem, levels of accuracy

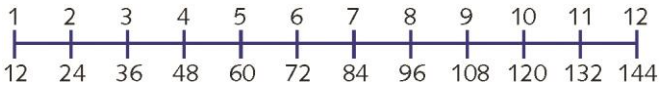
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

- solve problems involving division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates

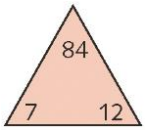
Mental strategies

- Continue to use models and images when necessary:
  - number lines



  - trios

7 × 12 = 84
12 × 7 = 84
84 ÷ 12 = 7
84 ÷ 7 = 12



  - multiplication square to 12 × 12/multiples of 10 multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

×	20	30	40	50	60	70	80	90	100	110	120
1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	240
3	60	90	120	150	180	210	240	270	300	330	360
4	80	120	160	200	240	280	320	360	400	440	480
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

- Continue to use the inverse relationship between multiplication and division

Year 5

DIVISION Continued

Mental strategies continued

- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number,
  486 ÷ 9 = (450 ÷ 9) + (36 ÷ 9)
  = 50 + 4
  = 54

Written methods

- Short division (including with remainders expressed as a whole number, fraction or decimal):
  - Divide numbers up to 4 digits by a one-digit number (HTO ÷ O/ThHTO ÷ O)
- Estimate and check the answer to a calculation

Formal written method of short division

Whole number remainder	Fraction remainder	Decimal remainder
279 ÷ 6	279 ÷ 6	279 ÷ 6
$\begin{array}{r} 46\text{ r }3 \\ 6\overline{)27^{3}9} \end{array}$	$\begin{array}{r} 46\frac{1}{2} \\ 6\overline{)27^{3}9} \end{array}$	$\begin{array}{r} 46.5 \\ 6\overline{)27^{3}9.^{3}0} \end{array}$

Children should describe what they are doing using phrases similar to the following: 'How many sixes divide into 270 so that the answer is a multiple of 10? (40) There are 40 sixes or 240, with 30 remaining.' The superscript 3 represents the 3 tens that are remaining after 6 has been divided into 270. It is written in front of the 9 to show that 39 now has to be divided by 6.

Children then ask: 'How many sixes in 39?' (6 remainder 3). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

Year 6

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- find 10, 100, 1000, 10 000 or 100 000 more or less than a given number
- round any whole number to a required degree of accuracy

DECIMALS

To add, subtract, multiply and divide successfully, pupils need to:

- read, write, order and compare numbers with up to three decimal places
- identify the value of each digit in numbers given to three decimal places
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:

- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- practise addition for larger numbers and decimals, using the formal written method of columnar addition
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:

- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Year 6

ADDITION Continued

Mental strategies

- Continue to use models and images when necessary:
  - decimals addition and subtraction tables

+	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0·1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1
0·2	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
0·3	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3
0·4	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4
0·5	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5
0·6	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6
0·7	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7
0·8	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8
0·9	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9
1	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2

+	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0·01	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11
0·02	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12
0·03	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13
0·04	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14
0·05	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15
0·06	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16
0·07	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17
0·08	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18
0·09	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19
0·1	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19	0·2

- Develop further the relationship between addition and subtraction
- Continue to apply knowledge of the commutative law, e.g.
  - put the larger number first and count on in steps of 1, 10, 100 or 1000
  - partition additions into hundreds, tens and ones, then recombine, e.g. 3356 + 257 = 3356 + 200 + 50 + 7
    = 3556 + 57
    = 3613
- Identify near doubles, using doubles already known, e.g. 5·7 + 5·8
- Add the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 9 + 7 = 16 and 0·09 + 0·07 = 0·16
- Use knowledge of the associative law when adding more than two numbers, e.g. 24 + 27 + 16 = (24 + 16) + 27
  = 40 + 27
  = 67

Written methods

Add numbers with more than four digits

- Add decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with
- different numbers of decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar addition

456 287 + 359 849

```
  456287
+ 359849
--------
  816136
  11111
```

57·486 + 45·378

```
  57·486
+ 45·378
--------
 102·864
   1  11
```

Carry digits are recorded below the line, using the words 'carry ten', 'carry one hundred', 'carry one thousand', … not 'carry one'.
Where appropriate, place value columns are labelled, e.g. TO·tht^{h}, to remind children of the value of each of the digits.

Year 6

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:

- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- practise subtraction for larger numbers and decimals, using the formal written method of columnar subtraction
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:

- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Mental strategies

- Continue to use models and images when necessary:
  - decimals addition and subtraction tables

+	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0·1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1
0·2	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
0·3	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3
0·4	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4
0·5	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5
0·6	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6
0·7	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7
0·8	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8
0·9	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9
1	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2

+	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0·01	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11
0·02	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12
0·03	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13
0·04	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14
0·05	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15
0·06	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16
0·07	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17
0·08	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18
0·09	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19
0·1	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19	0·2

- Develop further the relationship between addition and subtraction
- Calculate mentally a difference such as 23 004 – 18 998 by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 16 – 9 = 7 and 0·16 – 0·09 = 0·07
- Use partitioning, e.g. 4656 – 358 = 4656 – 300 – 50 – 8
  = 4356 – 58
  = 4298

Year 6

SUBTRACTION Continued

Written methods

- Subtract numbers with more than four digits
- Subtract decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

746 291 – 298 354 63·237 – 45·869

```
  6 13 15 12 8 11          5 12 11 12 17
   7 4 6  2 9 1             6 3 · 2 3 7
 – 2 9 8  3 5 4           – 4 5 · 8 6 9
   4 4 7  9 3 7             1 7 · 3 6 8
```

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens … and finally the hundreds of thousands. Refer to subtracting the tens, for example, by saying '8 tens subtract 5 tens', not '8 subtract 5'.

In the first example, the ones, tens, thousands and tens of thousands to be subtracted are all larger than all of the ones, tens, thousands and tens of thousands you are subtracting from.

The calculation begins by exchanging one of the 9 tens for 10 ones, crossing out the 9 and writing a superscript 8, and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 subtract 4.

You then calculate 8 tens subtract 5 tens.

Next, you exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 hundreds subtract 3 hundreds.

Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 5 to make 15 thousands. The calculation then becomes 15 thousands subtract 8 thousands.

Next, you exchange one of the 7 hundreds of thousands for 10 tens of thousands, crossing out the 7 and writing a superscript 6, and writing a superscript 1 in front of the 3 to make 13 tens of thousands. The calculation then becomes 13 tens of thousands subtract 9 tens of thousands.

Then, finally, 600 000 subtract 200 000.

Where appropriate, place value columns are labelled, e.g. TO·tht^{h}, to remind children of the value of each of the digits.

Year 6

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:

- consolidate recall of the multiplication facts for multiplication tables up to 12 × 12
- use known multiplication facts to derive related facts involving multiples of 10, 100 and 1000, and decimals, e.g. 70 × 80 = 5600, 0·8 × 6 = 4·8
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- continue to multiply whole numbers and those involving decimals by 10, 100 and 1000
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- continue to recognise commutativity in mental calculations
- practise multiplication for larger numbers, using the formal written method of short multiplication
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- multiply one-digit numbers with up to two decimal places by whole numbers
- multiply numbers with up to two decimal places by one- and two-digit whole numbers
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Year 6

MULTIPLICATION Continued

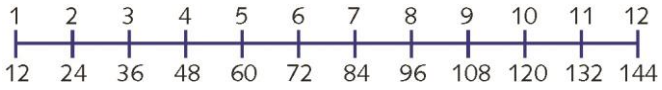
Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, multiplication to:

- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

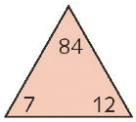
Mental strategies

- Continue to use models and images when necessary:
  - number lines



  - trios

$7 \times 12 = 84$
$12 \times 7 = 84$
$84 \div 12 = 7$
$84 \div 7 = 12$



  - multiplication square to 12 × 12/multiples of 10 multiplication square/decimals multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

×	20	30	40	50	60	70	80	90	100	110	120
1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	240
3	60	90	120	150	180	210	240	270	300	330	360
4	80	120	160	200	240	280	320	360	400	440	480
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

×	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
2	0·4	0·6	0·8	1	1·2	1·4	1·6	1·8	2	2·2	2·4
3	0·6	0·9	1·2	1·5	1·8	2·1	2·4	2·7	3	3·3	3·6
4	0·8	1·2	1·6	2	2·4	2·8	3·2	3·6	4	4·4	4·8
5	1	1·5	2	2·5	3	3·5	4	4·5	5	5·5	6
6	1·2	1·8	2·4	3	3·6	4·2	4·8	5·4	6	6·6	7·2
7	1·4	2·1	2·8	3·5	4·2	4·9	5·6	6·3	7	7·7	8·4
8	1·6	2·4	3·2	4	4·8	5·6	6·4	7·2	8	8·8	9·6
9	1·8	2·7	3·6	4·5	5·4	6·3	7·2	8·1	9	9·9	10·8
10	2	3	4	5	6	7	8	9	10	11	12
11	2·2	3·3	4·4	5·5	6·6	7·7	8·8	9·9	11	12·1	13·2
12	2·4	3·6	4·8	6	7·2	8·4	9·6	10·8	12	13·2	14·4

Year 6

MULTIPLICATION Continued

Mental strategies continued

- Continue to use the inverse relationship between multiplication and division
- Use related facts and doubling and halving:
  - double or halve the most significant digit first
  - to multiply by 25, multiply by 100 then divide by 4
  - double one number and halve the other
  - find the multiplication facts for the × 24 multiplication table by doubling the × 12 multiplication facts
  - squares of multiples of 10 to 100, e.g. 70 × 70, and the corresponding halves
  - doubles of decimals, e.g. 4·7 × 2, 0·63 × 2, and the corresponding halves
  - doubles of multiples of 10 to 1000, e.g. 830 × 2, and the corresponding halves
  - doubles of multiples of 100 to 10 000, e.g. 48 500 × 2, and the corresponding halves
- Use closely related facts:
  - multiply by 49 or 51 by multiplying by 50 and adjusting
  - develop the × 18 multiplication table by adding facts from the × 10 and × 8 multiplication tables
- Use factors, e.g. 9 × 18 = 9 × 6 × 3
- Use patterns of similar calculations, e.g. 8 × 6 = 48 and 0·8 × 6 = 4·8
- Continue to use and apply the commutative law
- Understand and use the associative law, e.g. 10·6 × 30 = 10·6 × (10 × 3)
  or = (10·6 × 10) × 3
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, or two two-digit numbers, and partitioning when multiplying a whole number or decimal by a one-digit number, e.g.

  285 × 63 = (200 × 63) + (80 × 63) + (5 × 63)
  = 12 600 + 5040 + 315
  = 17 955

  4·83 × 6 = (4 × 6) + (0·8 × 6) + (0·03 × 6)
  = 24 + 4·8 + 0·18
  = 28·98

Year 6

MULTIPLICATION Continued

Written methods

- Short multiplication (whole numbers):
  - Multiply multi-digit numbers up to 4 digits by a one-digit whole number
- Estimate and check the answer to a calculation

Formal written method of short multiplication

5643 × 8

```
  5 6 4 3
×       8
-------
 4 5 1 4 4
```

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

- Short multiplication (Decimals):
  - Multiply one-digit or two-digit numbers with up to two decimal places by a one-digit number
- Estimate and check the answer to a calculation

Grid method

4·83 × 6

×	4	0·8	0·03
6	24	4·8	0·18

= 28·98

Expanded written method

4·83 × 6 is equivalent to 483 × 6 ÷ 100

```
  4 · 8 3
×       6
-------
    1 8   (  3 × 6)
  4 8 0   ( 80 × 6)
2 4 0 0   (480 × 6)
---------
2 8 9 8
```

2898 ÷ 100 = 28·98

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns, e.g.

- when using the grid method and multiplying the tenths in 4·83 × 6 it is 'zero point eight multiplied by six', not 'eight multiplied by six'
- when using the expanded written method and multiplying the tens in 483 × 6 it is 'eighty multiplied by six, not 'eight multiplied by six'.

Although for both methods the relationship 8 × 6 should be stressed.

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

4·83 × 6 is equivalent to 483 × 6 ÷ 100

```
  4 8 3
×   4 1 6
---------
2 8 9 8
```

2898 ÷ 100 = 28·98

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 6

MULTIPLICATION Continued

Written methods continued

- Long multiplication (whole numbers):
  - Multiply multi-digit numbers up to four digits by a two-digit number (TO × TO / HTO × TO)
- Estimate and check the answer to a calculation

Grid method

285 × 63

×	200	80	5
60	12 000	4800	300
3	600	240	15

17 100 →
\+ 855 →
17 955 →

```
  285
×  63
17100
  855
17955
```

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 285 is multiplied by 60 (using knowledge of 285 x 6), then 285 is multiplied by 3, and finally the two products are added together.

Expanded written method

Multiplying the most significant digit first

```
  285
×  63
17 5 13 0 0   (285 × 60)
  8 2 5 1 5   (285 ×  3)
 1 7 9 5 5
```

Multiplying the least significant digit first

```
  285
×  63
  8 2 5 1 5   (285 ×  3)
17 5 13 0 0   (285 × 60)
 1 7 9 5 5
```

The grid method leads to the expanded written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of long multiplication

285 × 63

```
  285
×  63
  8 2 5 1 5
17 5 13 0 0
 1 7 9 5 5
```

The expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 6

MULTIPLICATION Continued

Written methods continued

- Long multiplication (Decimals):
  - Multiply one-digit numbers with up to two decimal places by a two-digit number
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Grid method

7·56 × 34

×	7	0·5	0·06
30	210	15	1·8
4	28	2	0·24

226·80
+ 30·24
257·04
1

The first step is to show all of the calculations involved, e.g. 7·56 is multiplied by 30 (using knowledge of 756 x 3), then 7·56 is multiplied by 4, and finally the two products are added together.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method

Multiplying the most significant digit first

7·56 × 34 is equivalent to 756 × 34 ÷ 100

```
   7 5 6
 ×   3 4
 2 2 16 18 0   (756 × 30)
   3 02 24     (756 ×  4)
 2 5 7 0 4
     1
```

25 704 ÷ 100 = 257·04

Multiplying the least significant digit first

7·56 × 34 is equivalent to 756 × 34 ÷ 100

```
   7 5 6
 ×   3 4
   3 02 24     (756 ×  4)
 2 2 16 18 0   (756 × 30)
 2 5 7 0 4
     1
```

25 704 ÷ 100 = 257·04

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of long multiplication

7·56 × 34 is equivalent to 756 × 34 ÷ 100

```
   7 5 6
 ×   3 4
   3 02 24
 2 2 16 18 0
 2 5 7 0 4
     1
```

25 704 ÷ 100 = 257·04

The expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 6

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:

- consolidate recall of the division facts for multiplication tables up to 12 × 12
- use known division facts to derive related facts involving multiples of 10, 100 and 1000, and decimals, e.g. 6300 ÷ 90 = 70, 6·3 ÷ 9 = 0·7
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- continue to divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- practise division for larger number, using the formal written method of short division
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- divide numbers with up to two decimal places by one- and two-digit whole numbers
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, division to:

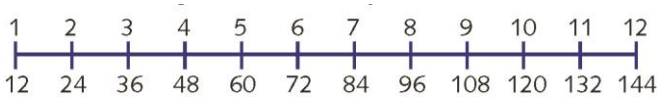
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Year 6

DIVISION Continued

Mental strategies

- Continue to use models and images when necessary:
  - number lines



  - multiplication square to 12 × 12/multiples of 10 multiplication square/decimals multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

×	20	30	40	50	60	70	80	90	100	110	120
1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	240
3	60	90	120	150	180	210	240	270	300	330	360
4	80	120	160	200	240	280	320	360	400	440	480
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

×	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
2	0·4	0·6	0·8	1	1·2	1·4	1·6	1·8	2	2·2	2·4
3	0·6	0·9	1·2	1·5	1·8	2·1	2·4	2·7	3	3·3	3·6
4	0·8	1·2	1·6	2	2·4	2·8	3·2	3·6	4	4·4	4·8
5	1	1·5	2	2·5	3	3·5	4	4·5	5	5·5	6
6	1·2	1·8	2·4	3	3·6	4·2	4·8	5·4	6	6·6	7·2
7	1·4	2·1	2·8	3·5	4·2	4·9	5·6	6·3	7	7·7	8·4
8	1·6	2·4	3·2	4	4·8	5·6	6·4	7·2	8	8·8	9·6
9	1·8	2·7	3·6	4·5	5·4	6·3	7·2	8·1	9	9·9	10·9
10	2	3	4	5	6	7	8	9	10	11	12
11	2·2	3·3	4·4	5·5	6·6	7·7	8·8	9·9	11	12·1	13·2
12	2·4	3·6	4·8	6	7·2	8·4	9·6	10·8	12	13·2	14·4

- Continue to use the inverse relationship between multiplication and division
- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number, e.g.
  486 ÷ 9 = (450 ÷ 9) + (36 ÷ 9)
  = 50 + 4
  = 54

Year 6

DIVISION Continued

Written methods

- Short division, including with remainders expressed as a whole number, fraction or decimal (whole numbers)
  - Divide numbers up to 4 digits by a one-digit number (HTO ÷ O/ThHTO ÷ O)
- Estimate and check the answer to a calculation

Formal written method of short division

Whole number remainder

1838 ÷ 8

$$\begin{array}{r} 2\ 2\ 9\ r\ 6 \\ 8\overline{)\,1\ 8^{2}3^{7}8} \end{array}$$

Fraction remainder

1838 ÷ 8

$$\begin{array}{r} 2\ 2\ 9\ \frac{3}{4} \\ 8\overline{)\,1\ 8^{2}3^{7}8} \end{array}$$

Decimal remainder

1838 ÷ 8

$$\begin{array}{r} 2\ 2\ 9\ .\ 7\ 5 \\ 8\overline{)\,1\ 8^{2}3^{7}8\ .^{6}0^{4}0} \end{array}$$

Children should describe what they are doing using phrases similar to the following: 'How many eights divide into 1800 so that the answer is a multiple of 100?' (200) There are 200 eights or 1600, with 200 remaining. The superscript 2 represents the 2 hundreds that are remaining after 8 has been divided into 1800. It is written in front of the 3 to show that a total of 23 tens (230) now have to be divided by eight.

Children then ask: 'How many eights divide into 230 so that the answer is a multiple of 10?' (20) There are 20 eights or 160, with 70 remaining. The superscript 7 represents the 7 tens that are remaining after 8 has been divided into 230. It is written in front of the 8 to show that 78 now has to be divided by 8.

Children then ask: 'How many eights in 78?' (9 remainder 6). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

- Short division (Decimals)
  - Divide numbers with up to two decimal places by a one-digit number (O·th ÷ O/TO·th ÷ O)
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

$$\begin{array}{r} \mathbf{7\ .\ 5\ 6} \\ \mathbf{6\overline{)\,4\ 5\ .^{3}3^{3}6}} \end{array}$$

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

45·36 ÷ 6 is equivalent to 4536 ÷ 6 ÷ 100

$$\begin{array}{r} \mathbf{7\ 5\ 6} \\ \mathbf{6\overline{)\,4\ 5^{3}3^{3}6}} \end{array}$$

756 ÷ 100 = 7·56

Phrases similar to those above for short division of whole numbers should be used for short division involving decimals.

An emphasis should be placed on recognising the value of each of the digits in the dividend.

- Long division, including with remainders expressed as a whole number, fraction or decimal (Whole numbers)
  - Divide numbers up to 4 digits by a two-digit number (HTO ÷ TO/ThHTO ÷ TO)
- Estimate and check the answer to a calculation

Expanded written method of long division

$$\begin{array}{r} 3\ 2\ 4\ r\ 4 \\ 18\overline{)\,5\ 8\ 3\ 6} \\ -\ 5\ 4\ 0\ 0 \quad (300 \times 18) \\ \hline 4\ 3\ 6 \\ -\ 3\ 6\ 0 \quad (20 \times 18) \\ \hline 0\ 7\ 6 \\ -\ 0\ 7\ 2 \quad (4 \times 18) \\ \hline 4 \end{array}$$

5836 ÷ 18 = 324 r 4 or $324\frac{2}{9}$

Formal written method of long division

$$\begin{array}{r} 3\ 2\ 4\ r\ 4 \\ 18\overline{)\,5\ 8\ 3\ 6} \\ -\ 5\ 4 \\ \hline 4\ 3 \\ -\ 3\ 6 \\ \hline 7\ 6 \\ -\ 7\ 2 \\ \hline 4 \end{array}$$

5836 ÷ 18 = 324 r 4 or $324\frac{2}{9}$

The amount of time that should be spent teaching and practising the expanded written method of long division will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.

Year 6

DIVISION Continued

Written methods continued

- Long division (Decimals)
  - Divide numbers with up to two decimal places by a two-digit whole number (TO·th ÷ TO)
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Expanded written method of long division

58·32 ÷ 18

```
      3 · 2 4
18) 5 8 · 3 2
  − 5 4 · 0 0   (  3 × 18)
    4 13 3 2
  −   3 · 6 0   ( 0.2 × 18)
      0 · 7 2
  −   0 · 7 2   (0.04 × 18)
      0 · 0 0
```

Formal written method of long division

58·32 ÷ 18

```
      3 · 2 4
18) 5 8 · 3 2
  − 5 4 ·  ↓  |
      4 · 3   ↓
  −   3 · 6
      0 · 7 2
  −   0 · 7 2
            0
```

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e Method 1) will depend on how secure the children are in their recall of multiplication and division facts, including involving decimals with up to two decimal places, with subtracting whole and decimal numbers mentally, and in their understanding of place value.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method of long division

58·32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

```
      3 2 4
18) 5 8 3 2
  − 5 4 0 0   (300 × 18)
    4 13 3 2
  −   3 6 0   ( 20 × 18)
      0 7 2
  −   0 7 2   (  4 × 18)
            0
```

324 ÷ 100 = 3.24

Formal written method of long division

58·32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

```
      3 2 4
18) 5 8 3 2
  − 5 4  ↓  |
      4 3   ↓
  −   3 6
        7 2
  −     7 2
            0
```

324 ÷ 100 = 3.24

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e. Method 2) will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.